

M2-brane superalgebra from Bagger-Lambert theory

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ABSTRACT: It is known that the M2-brane worldvolume superalgebra includes two p -form central charges that encode the M-theory intersections involving M2-branes. In this paper we show by explicit computation that the Bagger-Lambert Lagrangian realizes the M2-brane superalgebra, including also the central extensions. Solitons of the Bagger-Lambert theory, that are interpreted as worldvolume realizations of intersecting branes, are shown to saturate a BPS-bound given in terms of the corresponding central charge.

KEYWORDS: Extended Supersymmetry, M-Theory.

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1. Introduction and discussion

Brane intersections can be described as solitons of the worldvolume theory of one of the constituents of the intersecting system [1, 2]. In particular, quarter-BPS intersections appear on the worldvolume as half-BPS solitons and the spacetime interpretation relies on the fact that the worldvolume scalars encode the brane embedding.

Many M-branes systems in M-theory have been studied using this approach. For instance, a stack of M2-branes ending on an M5-brane is associated to a self-dual string soliton on the M5-brane worldvolume [3] and the M5-M5 intersection can be described as a 3-brane vortex on the worldvolume of one of the M5-branes [4]. In a similar way, the M2-M2 intersection can be described as a 0-brane vortex on the worldvolume of one of the M2-branes [1, 5]. All these examples mentioned are the worldvolume realization of previously studied quarter-BPS intersecting systems [6–9].

In a recent paper [10], Bagger and Lambert have proposed a Lagrangian to describe the low energy dynamics of a stack of coincident M2-branes (see also the work by Gustavsson [11]). Their model, that incorporates insights from previous papers [12, 13], includes half-BPS fuzzy 3-sphere solitons. This solutions were argued by Basu and Harvey [14] to provide the M2-branes worldvolume description of the multiple M2-branes ending on an M5-brane, generalizing a similar mechanism studied for the D1-D3 system [15]. The

Bagger-Lambert theory is a 3-dimensional $\mathcal{N} = 8$ supersymmetric field theory, based on a novel algebraic structure, dubbed 3-algebra. Explicit examples of 3-algebras has been recently constructed in [16–18] starting from ordinary Lie algebras and considering a Lorentzian scalar product (see also [19, 20]). The fact that the scalar product is not positive-definite permit to avoid a no-go theorem discussed in [21, 22]. Other algebraic structures have been considered in [23–27]. The Bagger-Lambert theory was shown to be conformal invariant in [28] and the moduli space was discussed in [29–33]. The maximally supersymmetric deformation of the theory was constructed in [34, 35] (see also [36]), for other deformations of the theory see [37–40]. In [41] the Bagger-Lambert theory is derived applying the embedding tensor methods and in [42, 43] the reduction to the theory of multiple D2-branes is discussed. Other recent developments are in [44–48].

It was shown in [49] that the spacetime interpretation of the worldvolume solitons can be deduced also from the worldvolume supersymmetry algebra. For the case of the M2-brane the worldvolume supersymmetry algebra is given by the maximal central extension of the 3-dimensional $\mathcal{N} = 8$ super-Poincare algebra [49]. The anticommutator is given by

$$\{Q_{\hat{\alpha}}^p, Q_{\hat{\beta}}^q\} = -2P_{\mu}(\hat{\gamma}^{\mu}\hat{\gamma}^0)_{\hat{\alpha}\hat{\beta}}\delta^{pq} + Z^{[pq]}\varepsilon_{\hat{\alpha}\hat{\beta}} + Z_{\mu}^{(pq)}(\hat{\gamma}^{\mu}\hat{\gamma}^0)_{\hat{\alpha}\hat{\beta}} \quad (1.1)$$

where $Q_{\hat{\alpha}}^p$ are the eight 3-dimensional Majorana spinor supercharges and $Z^{[pq]}, Z_{\mu}^{(pq)}$ are the 0-form and the 1-form worldvolume central charges. $p, q = 1, \dots, 8$ are the indices of the $SO(8)$ automorphism group and the supercharges transform as chiral spinors of $SO(8)$. Due to the triality relation of $SO(8)$, we can consider the supercharges to transform in the vector representation of $SO(8)$ and thus we can interpret the automorphism group $SO(8)$ as the rotation group in the eight directions transverse to the M2-branes. The 0-form $Z^{[pq]}$ is in the **28** representation of $SO(8)$ and it can be thought as a 2-form in the transverse space. This central charge is associated with M2-branes that are intersecting the original M2-branes along the time direction, a quarter-BPS system studied in [6]. The 1-form $Z_{\mu}^{(pq)}$ is in the **35**⁺ of $SO(8)$ and it is a self-dual 4-form in the transverse space. This implies that the 1-form charge is associated to the quarter-BPS M2-M5 system [9, 7].

We have seen that the M2-brane superalgebra, correctly incorporates all the possible quarter-BPS intersections between the M2-branes and the other M-branes of M-theory.¹ This implies that a complete M2-branes worldvolume theory should realize the M2-brane superalgebra (1.1), including also the central charges.

In this paper, we verify by explicit computation that the Bagger-Lambert theory does realize the M2-brane superalgebra (1.1). The central charges that we obtain are given by

$$\begin{aligned} Z^{[pq]} &= - \int d^2\sigma \partial_i \text{Tr}(X^I, D_j X^J) \varepsilon^{ij} (\gamma^{IJ})^{pq} \\ Z_{\mu}^{(pq)} &= - \frac{1}{12} \int d^2\sigma \partial_i \text{Tr}(X^I, [X^J, X^K, X^M]) \varepsilon^{0i}{}_{\mu} (\gamma^{IJKM})^{pq}. \end{aligned} \quad (1.2)$$

We note that the 0-form charge $Z^{[pq]}$ is the natural generalization of the charge computed in [5] using the BPS-bound for the vortex solution in the single M2-brane theory. The

¹In the worldvolume description, these intersections are half-BPS solitons.

1-form instead relies on the non-abelian nature of the scalar fields in the Bagger-Lambert theory and it vanishes in the limit where the stack of multiple M2-branes reduces to a single M2-brane. This is consistent with the fact that the M2-M5 intersection cannot be seen on the worldvolume of a single M2-brane. Indeed, given an intersection between branes with different dimensions, the worldvolume description of the system using the worldvolume of the lower dimensional brane is usually based on non-abelianity [15, 50].

We show that a vortex solution excites the 0-form central charge and the Basu-Harvey solution excites the 1-form central charge, in agreement with the interpretation of these solitons as the quarter-BPS M2-M2 intersection and the quarter-BPS M2-M5 intersection. The energy of this configurations is bounded below by the value of the corresponding central charge and the bound is saturated when the solitons are half-BPS. This is in agreement with the structure of the M2-brane superalgebra (1.1).

The rest of the paper is organized as follows. In section 2 we briefly review the Bagger-Lambert theory and we write down the supercurrent associated to the supersymmetry of the Lagrangian. This enables us to express the supercharges in terms of the fields of the theory. In section 3 we use the field theory realization of the supercharges to compute the central charges. In section 4 we analyze the vortex and the Basu-Harvey solitons and show that they are associated to central charges, in agreement with the interpretation of this solutions as the worldvolume realization of intersecting systems. Appendix A summarizes our notation and appendix B includes the proof of the conservation of the supercurrent. Appendix C contains technical details of the computation for the central charges.

2. The Bagger-Lambert theory

2.1 The lagrangian

We start reviewing the Lagrangian proposed by Bagger and Lambert [10] as the low energy effective theory for multiple coincident M2-branes. In this model, the transverse fluctuations of the membranes are described by eight scalar fields X^I , where $I = 3, \dots, 11$ and the eight Spin(1, 2) worldvolume fermions are collected together in the spinor field Ψ . The Ψ is an 11-dimensional Majorana spinor satisfying the condition $\Gamma_{012}\Psi = -\Psi$ and thus it has sixteen independent real components².

These fields are valued in a 3-algebra \mathcal{A} [10] (see also [11]), i.e. $X^I = X_a^I T^a$ and $\Psi = \Psi_a T^a$ where T^a , $a = 1, \dots, \dim \mathcal{A}$ are the generators of \mathcal{A} . The 3-algebra is endowed with a 3-product

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d \tag{2.1}$$

where the structure constants satisfy the fundamental identity

$$f^{efg}{}_d f^{abc}{}_g = f^{efa}{}_g f^{bcg}{}_d + f^{efb}{}_g f^{cag}{}_d + f^{efc}{}_g f^{abg}{}_d. \tag{2.2}$$

The 3-algebra construction includes also a bilinear and non-degenerate scalar product $\text{Tr}(\cdot, \cdot)$ that defines a non-degenerate metric h^{ab}

$$h^{ab} \equiv \text{Tr}(T^a, T^b) \tag{2.3}$$

²We summarize our conventions in appendix A.

used to manipulate the algebra indices. The structure constants f^{abcd} are assumed to be totally antisymmetric in the indices.

The Bagger-Lambert theory includes also a non-propagating gauge vector field $A_{\mu ab}$ where $\mu = 0, 1, 2$ denotes the worldvolume coordinates. The dynamics is controlled by the Lagrangian

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}(D_\mu X^a)(D^\mu X_a) + \frac{i}{2}\bar{\Psi}^a\Gamma^\mu D_\mu\Psi_a + \frac{i}{4}\bar{\Psi}_b\Gamma_{IJ}X_c^IX_d^J\Psi_a f^{abcd} \\ & -V + \frac{1}{2}\varepsilon^{\mu\nu\lambda}\left(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}\right) \end{aligned} \quad (2.4)$$

where V is the potential

$$V = \frac{1}{12}f^{abcd}f^{efg}{}_d X_a^IX_b^JX_c^KX_e^IX_f^JX_g^K = \frac{1}{2\cdot 3!}\text{Tr}([X^I, X^J, X^K], [X^I, X^J, X^K]) \quad (2.5)$$

and the covariant derivative of a field Φ is defined by

$$(D_\mu\Phi)_a = \partial_\mu\Phi_a - \tilde{A}_\mu{}^b{}_a\Phi_b \quad (2.6)$$

where $\tilde{A}_\mu{}^b{}_a \equiv f^{cdb}{}_a A_{\mu cd}$. The (2.4) is invariant under the gauge transformations

$$\begin{aligned} \delta X_a^I &= \tilde{\Lambda}^b{}_a X_b^I \\ \delta\Psi_a &= \tilde{\Lambda}^b{}_a\Psi_b \\ \delta\tilde{A}_\mu{}^b{}_a &= \partial_\mu\tilde{\Lambda}^b{}_a - \tilde{\Lambda}^b{}_c\tilde{A}_\mu{}^c{}_a + \tilde{A}_\mu{}^b{}_c\tilde{\Lambda}^c{}_a \end{aligned} \quad (2.7)$$

where $\tilde{\Lambda}^b{}_a \equiv f^{cdb}{}_a\Lambda_{cd}$ and Λ_{cd} is the gauge parameter. The Lagrangian (2.4) is also invariant under the following supersymmetry variations

$$\begin{aligned} \delta_\epsilon X_a^I &= i\bar{\epsilon}\Gamma^I\Psi_a \\ \delta_\epsilon\Psi_a &= D_\mu X_a^I\Gamma^\mu\Gamma^I\epsilon - \frac{1}{6}X_b^IX_c^JX_d^K f^{bcd}{}_a\Gamma^{IJK}\epsilon \\ \delta_\epsilon\tilde{A}_\mu{}^b{}_a &= i\bar{\epsilon}\Gamma_\mu\Gamma_I X_c^I\Psi_d f^{cdb}{}_a \end{aligned} \quad (2.8)$$

where the supersymmetry parameter ϵ satisfies $\Gamma_{012}\epsilon = \epsilon$. The equations of motion are

$$\begin{aligned} \Gamma^\mu D_\mu\Psi_a + \frac{1}{2}\Gamma_{IJ}X_c^IX_d^J\Psi_b f^{cdb}{}_a &= 0 \\ D^2 X_a^I - \frac{i}{2}\bar{\Psi}_c\Gamma^IX_d^J\Psi_b f^{cdb}{}_a - \frac{\partial V}{\partial X^Ia} &= 0 \\ \tilde{F}_{\mu\nu}{}^b{}_a + \varepsilon_{\mu\nu\lambda}\left(X_c^JD^\lambda X_d^J + \frac{i}{2}\bar{\Psi}_c\Gamma^\lambda\Psi_d\right) f^{cdb}{}_a &= 0 \end{aligned} \quad (2.9)$$

where

$$\tilde{F}_{\mu\nu}{}^b{}_a = \partial_\nu\tilde{A}_\mu{}^b{}_a - \partial_\mu\tilde{A}_\nu{}^b{}_a - \tilde{A}_\mu{}^b{}_c\tilde{A}_\nu{}^c{}_a + \tilde{A}_\nu{}^b{}_c\tilde{A}_\mu{}^c{}_a. \quad (2.10)$$

The stress-energy tensor $T_{\mu\nu}$ can be computed in the usual way coupling the Bagger-Lambert theory to an external worldvolume metric and looking at the variation of the

action for an infinitesimal change of the metric. In the case where the fermions are set to zero, it results

$$T_{\mu\nu} = D_\mu X_a^I D_\nu X^{aI} - \eta_{\mu\nu} \left(\frac{1}{2} D_\rho X^{aI} D^\rho X_a^I + V \right). \quad (2.11)$$

We note that the Chern-Simons like term in (2.4) does not contribute to the stress-energy tensor. This is because this term is topological and does not depend on the world-volume metric.

2.2 Supercharges

Given the invariance of the Lagrangian (2.4) under the supersymmetry variations (2.8), the Noether theorem implies the existence of a conserved supercurrent J^μ given by

$$J^\mu = -D_\nu X_a^I \Gamma^\nu \Gamma^I \Gamma^\mu \Psi^a - \frac{1}{6} X_a^I X_b^J X_c^K f^{abcd} \Gamma^{IJK} \Gamma^\mu \Psi_d. \quad (2.12)$$

In appendix B we show that $\partial_\mu J^\mu = 0$. The supercharge is thus the integral over the spatial worldvolume coordinates of the timelike component of the supercurrent, i.e.

$$\begin{aligned} Q &= \int d^2\sigma J^0 \\ &= - \int d^2\sigma (D_\nu X_a^I \Gamma^\nu \Gamma^I \Gamma^0 \Psi^a + \frac{1}{6} X_a^I X_b^J X_c^K f^{abcd} \Gamma^{IJK} \Gamma^0 \Psi_d). \end{aligned} \quad (2.13)$$

Given that the mass dimensions of the fields in the Bagger-Lambert theory are $[X] = \frac{1}{2}$ and $[A] = [\Psi] = 1$, it follows that J^0 has mass dimension $[J^0] = \frac{5}{2}$. This gives $[Q] = \frac{1}{2}$, that is the right mass dimension for the supercharge. It is easy to check that the two terms on the right hand side of (2.12) are the only gauge invariant combinations of fields with the right mass dimension and with an uncontracted spinorial index.

The supercharge Q is the generator of the supersymmetry transformation, that means that the supersymmetry variation of a field Φ is given by $\delta_\epsilon \Phi = [\bar{\epsilon} Q, \Phi]$. More in details, for Grassman-even and Grassman-odd fields Φ_E and Φ_O we have

$$\delta_\epsilon \Phi_E = \bar{\epsilon}_\alpha [Q^\alpha, \Phi_E] \quad \delta_\epsilon \Phi_O^\beta = \bar{\epsilon}_\alpha \{Q^\alpha, \Phi_O^\beta\} \quad (2.14)$$

where we have explicitly shown the 11-dimension spinorial indices α and β . Using the canonical commutation relations, one can show that the (2.14) reproduce the supersymmetry variations of the Bagger-Lambert theory (2.8).

3. Central charges

In this section, we show that the supersymmetry algebra of the Bagger-Lambert theory includes two central charge forms, as expected for a theory describing M2-branes. These central extensions are computed here explicitly using the field realization of the supercharge Q given in (2.13) [51].³ In details, we consider the relation

$$\bar{\epsilon}_\alpha \{Q^\alpha, Q^\beta\} = \int d^2\sigma \bar{\epsilon}_\alpha \{Q^\alpha, J^{0\beta}(\sigma)\} = \int d^2\sigma \delta_\epsilon J^{0\beta}(\sigma) \quad (3.1)$$

³For a review, see for instance [52].

where in the last step we used the second of the equations (2.14). The supersymmetry variation of the zeroth component of the supercurrent $\delta_\epsilon J^0$ is computed in the appendix C. For the case where the spinors Ψ are set to zero, it is given by

$$\delta_\epsilon J^0 = -2T^0{}_\mu \Gamma^\mu \epsilon - \partial_i (X_a^I D_j X^{aJ} \varepsilon^{ij} \Gamma^{IJ} \epsilon) - \frac{1}{12} \partial_i (X_a^I X_b^J X_c^K X_d^M f^{bcda} \varepsilon^{0i}{}_\mu \Gamma^{IJKM} \Gamma^\mu \epsilon) \quad (3.2)$$

where $i = 1, 2$ labels the spatial worldvolume coordinates. From the expression (3.2) and the relation (3.1) we get

$$\begin{aligned} \{Q^\alpha, Q^\beta\} &= -2P_\mu (\Gamma^\mu \Gamma^0)^{\alpha\beta} - \int d^2\sigma \partial_i (X_a^I D_j X^{aJ} \varepsilon^{ij}) (\Gamma^{IJ} \Gamma^0)^{\alpha\beta} \\ &\quad - \frac{1}{12} \int d^2\sigma \partial_i (X_a^I X_b^J X_c^K X_d^M f^{bcda} \varepsilon^{0i}{}_\mu) (\Gamma^{IJKM} \Gamma^\mu \Gamma^0)^{\alpha\beta} \end{aligned} \quad (3.3)$$

where P^μ is the energy momentum vector defined as $P^\mu = \int d^2\sigma T^{0\mu}$.

3.1 Spinors decomposition

In order to better analyze the structure of the $\mathcal{N} = 8$ superalgebra, we need to write the anticommutator (3.3) in terms of 3-dimensional spinors. To this end, we decompose the Spin(1, 10) Dirac matrices in terms of Spin(1, 2) \otimes Spin(8) Dirac matrices. In details, we take

$$\Gamma^\mu = \hat{\gamma}^\mu \otimes \bar{\gamma}_9 \quad \text{and} \quad \Gamma^I = 1 \otimes \bar{\gamma}^I \quad (3.4)$$

where

$$\{\hat{\gamma}^\mu, \hat{\gamma}^\nu\} = 2\eta^{\mu\nu}, \quad \{\bar{\gamma}^I, \bar{\gamma}^J\} = 2\delta^{IJ}, \quad \bar{\gamma}_9 = \bar{\gamma}^3 \dots \bar{\gamma}^{10} \quad (3.5)$$

and it is easy to check that the matrices (3.4) satisfies the 11-dimensional Clifford algebra. The $\hat{\gamma}^\mu$ are 2×2 real matrices. Explicitly

$$\hat{\gamma}^0 = i\sigma_{\hat{\alpha}\hat{\beta}}^2 = \varepsilon_{\hat{\alpha}\hat{\beta}} \quad \hat{\gamma}^1 = \sigma_{\hat{\alpha}\hat{\beta}}^1 \quad \hat{\gamma}^2 = \sigma_{\hat{\alpha}\hat{\beta}}^3 \quad (3.6)$$

where the σ 's are Pauli matrices and $\hat{\alpha}, \hat{\beta} = 1, 2$ are 3-dimensional spinorial indices. The $\bar{\gamma}^I$ are 16×16 real matrices given by

$$\bar{\gamma}^I = \begin{pmatrix} 0 & \gamma_{\hat{p}\hat{p}}^I \\ \gamma_{\hat{q}\hat{q}}^I & 0 \end{pmatrix} \quad (3.7)$$

where $(\gamma_{\hat{p}\hat{p}}^I)^T = \gamma_{\hat{p}\hat{p}}^I$ are 8×8 real gamma matrices satisfying

$$\gamma_{\hat{p}\hat{p}}^I \gamma_{\hat{p}\hat{q}}^J + \gamma_{\hat{p}\hat{p}}^J \gamma_{\hat{p}\hat{q}}^I = 2\delta^{IJ} \delta_{\hat{p}\hat{q}} \quad \gamma_{\hat{p}\hat{p}}^I \gamma_{\hat{p}\hat{q}}^J + \gamma_{\hat{p}\hat{p}}^J \gamma_{\hat{p}\hat{q}}^I = 2\delta^{IJ} \delta_{\hat{p}\hat{q}}. \quad (3.8)$$

Given that $\Gamma_{012} = -\hat{\gamma}_{012} \otimes \bar{\gamma}_9 = -1 \otimes \bar{\gamma}_9$, spinors with definite Γ_{012} chirality, have a definite $\bar{\gamma}_9$ chirality.⁴

⁴In this representation $\bar{\gamma}_9 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Using the matrices decomposition just described and the fact that $\Gamma_{012}Q = Q$, the equation (3.3) can be written as

$$\begin{aligned} \{Q_{\hat{\alpha}}^p, Q_{\hat{\beta}}^q\} &= -2P_{\mu}(\hat{\gamma}^{\mu}\hat{\gamma}^0)_{\hat{\alpha}\hat{\beta}}\delta^{pq} - \int d^2\sigma\partial_i(X_a^I D_j X^{aJ}\varepsilon^{ij})(\gamma^{IJ})^{pq}\varepsilon_{\hat{\alpha}\hat{\beta}} \\ &\quad - \frac{1}{12} \int d^2\sigma\partial_i(X_a^I X_b^J X_c^K X_d^M f^{bcda}\varepsilon^{0i}_{\mu})(\gamma^{IJKM})^{pq}(\hat{\gamma}^{\mu}\hat{\gamma}^0)_{\hat{\alpha}\hat{\beta}} \end{aligned} \quad (3.9)$$

where $(\gamma^{IJ})_{pq} = \gamma_{pr}^I \gamma_{rq}^J$ and $(\gamma^{IJKM})_{pq} = \gamma_{pr}^I \gamma_{rs}^J \gamma_{st}^K \gamma_{tq}^M$.

Thus, we conclude that the Bagger-Lambert Lagrangian realizes the centrally extended 3-dimensional $\mathcal{N} = 8$ superalgebra

$$\{Q_{\hat{\alpha}}^p, Q_{\hat{\beta}}^q\} = -2P_{\mu}(\hat{\gamma}^{\mu}\hat{\gamma}^0)_{\hat{\alpha}\hat{\beta}}\delta^{pq} + Z^{[pq]}\varepsilon_{\hat{\alpha}\hat{\beta}} + Z_{\mu}^{(pq)}(\hat{\gamma}^{\mu}\hat{\gamma}^0)_{\hat{\alpha}\hat{\beta}} \quad (3.10)$$

where the central charges are given by

$$\begin{aligned} Z^{[pq]} &= - \int d^2\sigma\partial_i \text{Tr}(X^I, D_j X^J)\varepsilon^{ij}(\gamma^{IJ})^{pq} \\ Z_{\mu}^{(pq)} &= -\frac{1}{12} \int d^2\sigma\partial_i \text{Tr}(X^I, [X^J, X^K, X^M])\varepsilon^{0i}_{\mu}(\gamma^{IJKM})^{pq}. \end{aligned} \quad (3.11)$$

Using the property $(\gamma^{IJ})^{qp} = -(\gamma^{IJ})^{pq}$, $(\gamma^{IJKM})^{qp} = (\gamma^{IJKM})^{pq}$ it follows that the 0-form central charge is antisymmetric in p, q indices and the 1-form central charge is symmetric in p, q . Given that $\varepsilon_{\hat{\alpha}\hat{\beta}} = -\varepsilon_{\hat{\beta}\hat{\alpha}}$ and $(\hat{\gamma}^{\mu}\hat{\gamma}^0)_{\hat{\alpha}\hat{\beta}} = (\hat{\gamma}^{\mu}\hat{\gamma}^0)_{\hat{\beta}\hat{\alpha}}$ the right hand side of the (3.11) is correctly symmetric under the exchange $(p, \hat{\alpha}) \leftrightarrow (q, \hat{\beta})$.

The equations (3.11) give the field realization of the central charges of the extended 3-dimensional $\mathcal{N} = 8$ superalgebra. They are boundary terms and they are equal to zero for field configurations that are non-singular and topologically trivial. In the next section we will discuss half-BPS configurations that excite the central charges of the Bagger-Lambert theory.

4. Solitons of the Bagger-Lambert theory

4.1 Vortices

We consider vortex configurations [1, 5] where only the scalars X^3, X^4 and the gauge vector $\tilde{A}_{\nu}{}^b{}_a$ are excited. Given the interpretation of the Bagger-Lambert theory as a theory of coincident M2-branes, these configurations describe two stacks of membranes intersecting along the time direction⁵

$$\begin{array}{cccccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ \text{M2} & \text{X} & \text{X} & \text{X} & & & & & & & & \\ \text{M2} & \text{X} & & & \text{X} & \text{X} & & & & & & \end{array} \quad (4.1)$$

⁵This is the analog of the vortex like solution for $\mathcal{N} = 4$ SYM describing a surface operator interpreted as the intersection $D3 \cap D3 = R^2$ [53, 54].

It is convenient to introduce the complex worldvolume coordinates z and \bar{z}

$$z = \sigma^1 + i\sigma^2 \quad \bar{z} = \sigma^1 - i\sigma^2 \quad (4.2)$$

and the complex scalars Φ and $\bar{\Phi}$

$$\Phi = \frac{1}{2}(X_3 - iX_4) \quad \bar{\Phi} = \frac{1}{2}(X_3 + iX_4). \quad (4.3)$$

Thus, considering a configuration where only Φ , $\bar{\Phi}$ and $\tilde{A}_\mu{}^b{}_a$ are switched on, and such that $D_0\Phi = D_0\bar{\Phi} = 0$, the BPS conditions that follow from the supersymmetry variations (2.8) reduce to

$$D_z\Phi\Gamma^z\Gamma^\Phi\epsilon + D_{\bar{z}}\Phi\Gamma^{\bar{z}}\Gamma^\Phi\epsilon + D_z\bar{\Phi}\Gamma^z\Gamma^{\bar{\Phi}}\epsilon + D_{\bar{z}}\bar{\Phi}\Gamma^{\bar{z}}\Gamma^{\bar{\Phi}}\epsilon = 0 \quad (4.4)$$

where

$$\Gamma^\Phi = \Gamma^3 + i\Gamma^4 \quad \Gamma^{\bar{\Phi}} = \Gamma^3 - i\Gamma^4 \quad \Gamma^z = \Gamma^1 + i\Gamma^2 \quad \Gamma^{\bar{z}} = \Gamma^1 - i\Gamma^2. \quad (4.5)$$

For this configuration, the energy density is given by

$$\mathcal{H} = 4\text{Tr}(D_z\Phi, D_{\bar{z}}\bar{\Phi}) + 4\text{Tr}(D_{\bar{z}}\Phi, D_z\bar{\Phi}) = \frac{\mathcal{Z}^0}{2} + 8\text{Tr}(D_{\bar{z}}\Phi, D_z\bar{\Phi}) \quad (4.6)$$

where \mathcal{Z}^0 is the density of the 0-form central charge $Z^{[pq]}$ evaluated for this field configuration. Thus, considering a positive definite scalar product $\text{Tr}(\cdot, \cdot)$, it results $\mathcal{H} \geq \frac{\mathcal{Z}^0}{2}$ and the bound is saturated when

$$D_{\bar{z}}\Phi = D_z\bar{\Phi} = 0. \quad (4.7)$$

When this last condition is satisfied, it follows from the BPS equation (4.4) that the solution preserve the supersymmetries satisfying $\Gamma^z\Gamma^\Phi\epsilon = 0$ or equivalently $\Gamma^{1234}\epsilon = \epsilon$. Thus, for the case where the gauge field is equal to zero, i.e. $\tilde{A}_\mu{}^b{}_a = 0$ the vortex configuration given by

$$\Phi = \frac{c_a T^a}{z} \quad (4.8)$$

where c_a are arbitrary constants is a half-BPS state.⁶ The singularity in $z = 0$ excite the 0-form central charge $Z^{[pq]}$ (3.11), in agreement with the interpretation of the vortex solution as the brane intersection (4.1).

We now discuss the case where also the gauge vector is excited and to analyze this configuration we use the 3-algebra constructed in [16]. In this model, the 3-algebra indices a are split into $a = (0, \tilde{a}, \varphi)$ and the structure constants are given by

$$f^{0\tilde{a}\tilde{b}\tilde{c}} = f^{\varphi\tilde{a}\tilde{b}\tilde{c}} = C^{\tilde{a}\tilde{b}\tilde{c}}, \quad f^{0\varphi\tilde{a}\tilde{b}} = f^{\tilde{a}\tilde{b}\tilde{c}\tilde{d}} = 0 \quad (4.9)$$

where $C^{\tilde{a}\tilde{b}\tilde{c}}$ are the structure constants of a compact semi-simple Lie algebra satisfying the usual Jacobi identity. The structure constants (4.9) solve the fundamental identity (2.2)

⁶In the sense that it preserves half of the supersymmetries (2.8).

and they are totally antisymmetric. Following [16], we introduce null generators on the 3-algebra

$$T^\pm = \pm T^0 + T^\phi \quad (4.10)$$

and in this basis the structure constants become

$$f^{+\tilde{a}\tilde{b}\tilde{c}} = 2C^{\tilde{a}\tilde{b}\tilde{c}}, \quad f_{-\tilde{a}\tilde{b}\tilde{c}} = C_{\tilde{a}\tilde{b}\tilde{c}}, \quad f^{-\tilde{a}\tilde{b}\tilde{c}} = f_{+\tilde{a}\tilde{b}\tilde{c}} = 0. \quad (4.11)$$

The gauge vector A_μ^{ab} is decomposed as

$$A_\mu^{\tilde{a}} \equiv A_\mu^{-\tilde{a}}, \quad B_\mu^{\tilde{a}} \equiv \frac{1}{2}C^{\tilde{a}\tilde{b}\tilde{c}}A_{\mu\tilde{b}\tilde{c}}. \quad (4.12)$$

We consider a configuration where only the \tilde{a} components of the scalar field are excited, we call this field $\tilde{\Phi}$. Thus

$$\tilde{\Phi} = \frac{c_{\tilde{a}}T^{\tilde{a}}}{z}. \quad (4.13)$$

Taking $B_\mu = 0$, the equation (4.4) reduce to

$$\tilde{D}_z\tilde{\Phi}\Gamma^z\Gamma^\Phi\epsilon + \tilde{D}_{\bar{z}}\tilde{\Phi}\Gamma^{\bar{z}}\Gamma^\Phi\epsilon + \tilde{D}_z\tilde{\Phi}\Gamma^z\Gamma^{\bar{\Phi}}\epsilon + \tilde{D}_{\bar{z}}\tilde{\Phi}\Gamma^{\bar{z}}\Gamma^{\bar{\Phi}}\epsilon = 0 \quad (4.14)$$

where

$$\tilde{D}_\mu\tilde{\Phi}^{\tilde{a}} \equiv \partial_\mu\tilde{\Phi}^{\tilde{a}} + 2C^{\tilde{a}}_{\tilde{b}\tilde{c}}A_\mu^{\tilde{c}}\tilde{\Phi}^{\tilde{b}} \quad (4.15)$$

is the covariant derivative for a field in the adjoint representation of the Lie algebra with structure constants $C^{\tilde{a}\tilde{b}\tilde{c}}$. The energy density now is

$$\mathcal{H} = 4\text{Tr}(\tilde{D}_z\tilde{\Phi}, \tilde{D}_{\bar{z}}\tilde{\Phi}) + 4\text{Tr}(\tilde{D}_{\bar{z}}\tilde{\Phi}, \tilde{D}_z\tilde{\Phi}) = \frac{\mathcal{Z}^0}{2} + 8\text{Tr}(\tilde{D}_{\bar{z}}\tilde{\Phi}, \tilde{D}_z\tilde{\Phi}) \quad (4.16)$$

and given that [16] $\text{Tr}(T^{\tilde{a}}, T^{\tilde{b}}) = \delta^{\tilde{a}\tilde{b}}$, it results $\mathcal{H} \geq \frac{\mathcal{Z}^0}{2}$. The \mathcal{Z}^0 is the 0-form central charge evaluated for this solution and the BPS-bound is saturated when $\tilde{D}_{\bar{z}}\tilde{\Phi} = \tilde{D}_z\tilde{\Phi} = 0$. Thus, it follows that the configuration where only $\tilde{\Phi}$ and $\mathcal{A}_\mu = A_{\tilde{a}\mu}T^{\tilde{a}}$ are excited, is half-BPS if

$$[\tilde{\Phi}, \mathcal{A}_{\bar{z}}] = [\tilde{\Phi}, \mathcal{A}_z] = 0 \quad (4.17)$$

where $[\cdot, \cdot]$ is the usual Lie commutator. Also in this case, the preserved supersymmetries satisfy $\Gamma^z\Gamma^\Phi\epsilon = 0$ and this configuration excites the the 0-form central charge. This implies that with respect to the single M2-brane theory, the vortex solutions of the Bagger-Lambert theory includes extra degrees of freedom, given by the the components of the gauge vector that commute with the scalar fields.

4.2 Basu-Harvey solitons

To describe a stack of M2-branes ending on an M5-brane

$$\begin{array}{cccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ M2 & X & X & X & & & & & & & & \\ M5 & X & X & & X & X & X & X & & & & \end{array} \quad (4.18)$$

it is necessary to switch on the X^3, X^4, X^5, X^6 scalar fields [14]. Given that these fields depend only on the worldvolume coordinate σ^2 , the BPS condition is [12]

$$\frac{dX^A}{d\sigma^2} \Gamma^A \Gamma^2 \epsilon - \frac{1}{6} \varepsilon^{BCDA} \Gamma^A [X^B, X^C, X^D] \Gamma^{3456} \epsilon = 0 \quad (4.19)$$

where $A, B, C, D = 3, 4, 5, 6$ and we used $\varepsilon^{ABCD} \Gamma^D = -\Gamma^{ABC} \Gamma_{3456}$. For this field configuration the energy density is given by

$$\mathcal{H} = \frac{1}{2} \text{Tr}(\partial_2 X^A, \partial_2 X^A) + \frac{1}{12} \text{Tr}([X^A, X^B, X^C], [X^A, X^B, X^C]). \quad (4.20)$$

Following [29], we write the potential as

$$V(X) = \frac{1}{2} \text{Tr} \left(\frac{\partial W}{\partial X^A}, \frac{\partial W}{\partial X^A} \right) \quad (4.21)$$

where

$$W = \frac{1}{24} \varepsilon^{ABCD} \text{Tr}(X^A, [X^B, X^C, X^D]). \quad (4.22)$$

Thus

$$\begin{aligned} \mathcal{H} &= \frac{1}{2} \text{Tr} \left(\partial_2 X^A + \frac{\partial W}{\partial X^A}, \partial_2 X^A + \frac{\partial W}{\partial X^A} \right) - \text{Tr} \left(\partial_2 X^A, \frac{\partial W}{\partial X^A} \right) \\ &= \frac{1}{2} \text{Tr} \left(\partial_2 X^A + \frac{\partial W}{\partial X^A}, \partial_2 X^A + \frac{\partial W}{\partial X^A} \right) + \frac{\mathcal{Z}^1}{2} \end{aligned} \quad (4.23)$$

where \mathcal{Z}^1 is the density of $Z_\mu^{(pq)}$, the 1-form central charge. Thus, for this field configuration $\mathcal{H} \geq \frac{\mathcal{Z}^1}{2}$ and the bound is saturated when

$$\frac{dX^A}{d\sigma^2} - \frac{1}{6} \varepsilon^{BCDA} [X^B, X^C, X^D] = 0. \quad (4.24)$$

When the (4.24) is satisfied, it follows from (4.19) that the field configuration is half-BPS and the preserved supersymmetries satisfy $\Gamma^2 \epsilon = \Gamma^{3456} \epsilon$. This is the configuration proposed by Basu and Harvey as the M2-brane worldvolume soliton describing the branes system (4.18). In this section we have verified that the central charge associated to this state is the the 1-form central charge, i.e. the central charge associated to the M2-M5 intersection.

Acknowledgments

We are grateful to Jaume Gomis for suggesting the problem and for useful discussions. We also would like to thank Joaquim Gomis and Shunji Matsuura for fruitful conversations. This research was supported by Perimeter Institute for Theoretical Physics. Research at Perimeter Institute is supported by the Government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Research and Innovation. We also acknowledge further support by an NSERC Discovery Grant.

A. Notation

We summarize here our notation. The indices are

$$\begin{aligned}
 \text{worldvolume coordinates :} & \quad \mu, \nu = 0, 1, 2 \\
 \text{spatial worldvolume coordinates :} & \quad i, j = 1, 2 \\
 \text{transverse space coordinates :} & \quad I, J = 3, \dots, 10 \\
 \text{Spin}(1, 10) \text{ spinorial indices :} & \quad \alpha, \beta = 1, \dots, 32 \\
 \text{Spin}(1, 2) \text{ spinorial indices :} & \quad \hat{\alpha}, \hat{\beta} = 1, 2 \\
 \text{Spin}(8) \text{ chiral spinorial indices :} & \quad p, q, \dot{p}, \dot{q} = 1, \dots, 8 \\
 \mathcal{A} \text{ algebra indices :} & \quad a, b = 1, \dots, \dim \mathcal{A}
 \end{aligned} \tag{A.1}$$

The Dirac matrices Γ are a representation of the 11-dimensional Clifford algebra, i.e. given $m, n = 0, \dots, 10$ it results

$$\{\Gamma^m, \Gamma^n\} = 2\eta^{mn} \tag{A.2}$$

and

$$C^T = -C \quad \Gamma_m^T = -C\Gamma_m C^{-1}. \tag{A.3}$$

We take Γ_m to be real matrices and $C = \Gamma^0$. The 11-dimensional spinors are Majorana (real) spinors with definite chirality respect to Γ_{012} . Thus, they have 16 independent real components.

B. Supercurrent conservation

We now show that the supercurrent (2.12) is conserved. An easy computation gives

$$\begin{aligned}
 \partial_\mu J^\mu &= -\partial_\mu (D_\nu X_a^I) \Gamma^\nu \Gamma^I \Gamma^\mu \Psi^a - D_\nu X_a^I \Gamma^\nu \Gamma^I \Gamma^\mu \partial_\mu \Psi^a \\
 &\quad - \frac{1}{2} \partial_\mu X_a^I X_b^J X_c^K f^{abcd} \Gamma^{IJK} \Gamma^\mu \Psi_d \\
 &\quad - \frac{1}{6} X_a^I X_b^J X_c^K f^{abcd} \Gamma^{IJK} \Gamma^\mu \partial_\mu \Psi_d.
 \end{aligned} \tag{B.1}$$

Using the fundamental identity (2.2) the previous equation can be rewritten as

$$\begin{aligned}
 \partial_\mu J^\mu &= -(D_\mu D_\nu X_a^I) \Gamma^\nu \Gamma^I \Gamma^\mu \Psi^a - D_\nu X_a^I \Gamma^\nu \Gamma^I \Gamma^\mu D_\mu \Psi^a \\
 &\quad - \frac{1}{2} D_\mu X_a^I X_b^J X_c^K f^{abcd} \Gamma^{IJK} \Gamma^\mu \Psi_d \\
 &\quad - \frac{1}{6} X_a^I X_b^J X_c^K f^{abcd} \Gamma^{IJK} \Gamma^\mu D_\mu \Psi_d.
 \end{aligned} \tag{B.2}$$

Inserting the equations of motion (2.9) and using the identity

$$\bar{\Psi}_c \Gamma^{IJ} \Psi_b \Gamma^I \Psi_a X_d^J f^{cdba} = -\bar{\Psi}_c \Gamma_\mu \Psi_b \Gamma^\mu \Gamma^J \Psi_a X_d^J f^{cdba}, \tag{B.3}$$

the right hand side of the (B.2) results to be equal to zero.

C. Supersymmetry variation of J^0

In this appendix we compute the supersymmetry variation of J^0 , the zeroth component of the supercurrent (2.12). Considering the ansatz $\Psi = 0$ we get

$$\begin{aligned} \delta_\epsilon J^0 = & -D_\mu X_a^I D_\nu X^{aJ} \Gamma^\mu \Gamma^I \Gamma^0 \Gamma^\nu \Gamma^J \epsilon + \frac{1}{6} D_\mu X_a^I X_b^J X_c^K X_d^M f^{bcda} \Gamma^\mu \Gamma^I \Gamma^0 \Gamma^{JKM} \epsilon \\ & - \frac{1}{6} D_\mu X_a^I X_b^J X_c^K X_d^M f^{bcda} \Gamma^{JKM} \Gamma^0 \Gamma^\mu \Gamma^I \epsilon \\ & + \frac{1}{36} X_a^I X_b^J X_c^K X_e^L X_f^M X_g^N f^{abcd} f^{efg} \Gamma^{IJK} \Gamma^0 \Gamma^{LMN} \epsilon. \end{aligned} \quad (C.1)$$

We note that the right hand side of (C.1) contains one term with two covariant derivatives, two terms with one covariant derivative and one term without covariant derivatives. Let's look first at the term with two covariant derivatives. Using the identity

$$\begin{aligned} -\Gamma^\mu \Gamma^I \Gamma^0 \Gamma^\nu \Gamma^J = & \Gamma^0 \Gamma^{\mu\nu} \Gamma^{IJ} + \Gamma^0 \Gamma^{\mu\nu} \delta^{IJ} + \Gamma^0 \eta^{\mu\nu} \Gamma^{IJ} \\ & + \Gamma^0 \eta^{\mu\nu} \delta^{IJ} - 2\eta^{\mu 0} \Gamma^\nu \delta^{IJ} - 2\eta^{\mu 0} \Gamma^\nu \Gamma^{IJ} \end{aligned} \quad (C.2)$$

we have

$$\begin{aligned} -D_\mu X_a^I D_\nu X^{aJ} \Gamma^\mu \Gamma^I \Gamma^0 \Gamma^\nu \Gamma^J \epsilon = & (D_0 X_a^I D_0 X^{aI} + D_i X_a^I D_i X^{aI}) \Gamma^0 \epsilon + 2D_0 X_a^I D_i X^{aI} \Gamma^i \epsilon \\ & + D_i X_a^I D_j X^{aJ} \Gamma^0 \Gamma^{ij} \Gamma^{IJ} \epsilon. \end{aligned} \quad (C.3)$$

The two terms with one covariant derivative can be rearranged using the identity

$$-\Gamma^\mu \Gamma^0 \Gamma^I \Gamma^{JKM} - \Gamma^0 \Gamma^\mu \Gamma^{JKM} \Gamma^I = 2\eta^{\mu i} \Gamma^0 \Gamma^i \Gamma^{IJKM} - 6\eta^{\mu 0} \delta^I [J \Gamma^{KM}] \quad (C.4)$$

and the last of the equations of motion (2.9). Thus we get

$$\begin{aligned} & + \frac{1}{6} D_\mu X_a^I X_b^J X_c^K X_d^M f^{bcda} \Gamma^\mu \Gamma^I \Gamma^0 \Gamma^{JKM} \epsilon \\ & - \frac{1}{6} D_\mu X_a^I X_b^J X_c^K X_d^M f^{bcda} \Gamma^{JKM} \Gamma^0 \Gamma^\mu \Gamma^I \epsilon = \\ & + \frac{1}{3} D_i X_a^I X_b^J X_c^K X_d^M f^{bcda} \Gamma^0 \Gamma^i \Gamma^{IJKM} \epsilon + \frac{1}{2} \varepsilon_{ij} \tilde{F}^{ijcd} X_c^I X_d^J \Gamma^{IJ} \epsilon. \end{aligned} \quad (C.5)$$

Using the fundamental identity (2.2) one can show that

$$\tilde{A}_i{}^g{}_a X_g^I X_b^J X_c^K X_d^M f^{bcda} \Gamma^{IJKM} = 0 \quad (C.6)$$

thus the (C.5) can be rewritten as

$$\begin{aligned} & + \frac{1}{6} D_\mu X_a^I X_b^J X_c^K X_d^M f^{bcda} \Gamma^\mu \Gamma^I \Gamma^0 \Gamma^{JKM} \epsilon \\ & - \frac{1}{6} D_\mu X_a^I X_b^J X_c^K X_d^M f^{bcda} \Gamma^{JKM} \Gamma^0 \Gamma^\mu \Gamma^I \epsilon = \\ & + \frac{1}{12} \partial_i (X_a^I X_b^J X_c^K X_d^M f^{bcda} \Gamma^0 \Gamma^i \Gamma^{IJKM} \epsilon) + \frac{1}{2} \varepsilon_{ij} \tilde{F}^{ijcd} X_c^I X_d^J \Gamma^{IJ} \epsilon. \end{aligned} \quad (C.7)$$

The term without covariant derivatives can be simplified using the expression

$$\Gamma^{IJK}\Gamma^{LMN} = \Gamma^{IJKLMN} + 9\Gamma^{[IJ}{}_{[MN}\delta_L^K]} + 18\Gamma^{[I}{}_{[N}\delta_L^K}\delta_M^J]} + 6\delta_{[N}^{[I}\delta_M^J}\delta_L^K]} \quad (\text{C.8})$$

and the property of the f^{abcd} structure constants. We get

$$\begin{aligned} \frac{1}{36}X_a^IX_b^JX_c^KX_e^LX_f^MX_g^Nf^{abcd}f^{efg}{}_d\Gamma^{IJK}\Gamma^0\Gamma^{LMN}\epsilon &= \frac{1}{6}\Gamma^0\epsilon X_a^IX_b^JX_c^KX_e^LX_f^MX_g^Kf^{abcd}f^{efg}{}_d \\ &= 2\Gamma^0\epsilon V \end{aligned} \quad (\text{C.9})$$

where V is the potential defined in (2.5).

Collecting all the pieces together we have

$$\begin{aligned} \delta_\epsilon J^0 &= (D_0X_a^ID_0X^{aI} + D_iX_a^ID_iX^{aI} + 2V)\Gamma^0\epsilon + 2D_0X_a^ID_iX^{aI}\Gamma^i\epsilon + \\ &\quad -\partial_i(X_a^ID_jX^{aJ}\epsilon^{ij}\Gamma^{IJ}\epsilon) + \frac{1}{12}\partial_i(X_a^IX_b^JX_c^KX_d^Mf^{bcda}\Gamma^0\Gamma^i\Gamma^{JKM}\epsilon) \end{aligned} \quad (\text{C.10})$$

Considering the ansatz $\Psi = 0$, the components of the stress-energy tensor (2.11) are

$$\begin{aligned} T_{00} &= \frac{1}{2}D_0X_a^ID_0X^{aI} + \frac{1}{2}D_iX_a^ID_iX^{aI} + V \\ T_{0i} &= D_0X_a^ID_iX^{aI} \end{aligned} \quad (\text{C.11})$$

Using the (C.11) and the identity $\Gamma^0\Gamma^i = -\epsilon^{ij}\Gamma^j\Gamma_{012}$ the (C.10) can be rewritten as

$$\delta_\epsilon J^0 = -2T^0{}_\mu\Gamma^\mu\epsilon - \partial_i(X_a^ID_jX^{aJ}\epsilon^{ij}\Gamma^{IJ}\epsilon) - \frac{1}{12}\partial_i(X_a^IX_b^JX_c^KX_d^Mf^{bcda}\epsilon^{0i}{}_\mu\Gamma^{IJKM}\Gamma^\mu\epsilon). \quad (\text{C.12})$$

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